LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

THIRD SEMESTER – November 2009

ST 3503/ST 3501/ST 3500 - STATISTICAL MATHEMATICS - II

Date & Time: 4/11/2009 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

SECTION – A

Answer ALL the questions (10 x 2 = 20 marks)

1. When do you say that a bounded function is Riemann integrable on [a, b]?
2. Given that X has the probability mass function p(x) = $\frac{1}{8}\ 3_{C_x}$ for x = 0, 1,2, 3, find the M.G.F. of X.
3. Prove that β(m, n) = β (n,m).
4. Prove that L(Cos at) = $\frac{s}{s^2+a^2}$.
5. Evaluate $\int_0^b \int_0^a xy\,(x-y)dx\ dy$.
6. Given f(x,y) = $\begin{cases} xe^{-x(1+y)}, & x \geq 0,\ y \geq 0 \\ 0, & other\ wise \end{cases}$. Find E (X Y).
7. Solve the equation $(D^2+1)y = 0$.
8. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} - 4\sqrt{\frac{dy}{dx}} = 5$.
9. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find the characteristic equation of A.
10. Show that the system of equations 3x – 4y = 2 ; 5x+2y = 12; - x+3y = 1 are consistent.

SECTION – B

Answer any FIVE questions. (5 x 8 = 40 marks)

11. Let f be a bounded function on the closed, bounded interval [a, b],
    Show that $f \in$ R[a,b] if and only if for each ∈>0 there exists a subdivision σ of [a,b] such that
    U [f ; σ] < L[f ; σ] +∈.....
12. A continuous random variable X has a p.d.f. given by f(x) = $\begin{cases} k\,x\,e^{-\lambda x}, & x > 0,\ \lambda > 0 \\ 0, & other\ wise \end{cases}$.
    Determine the constant k and find the mean and variance of X, when λ is a known constant.
13. Prove that the improper integral $\int_1^{\infty} \frac{dx}{n}$ diverges.

14. Find L $\left(\frac{1-e^t}{t}\right)$.

15. Given the joint density function of X and Y as f (x, y) =

$\frac{x}{2}\, e^{-y}, \quad 0 < x < 2, \quad y > 0$

$0\,, \quad other\ wise$.

Find the distribution of X+Y.

16. Solve $(D^2 - 2D + 3)\, y = x^3$.

17. Using Laplace transform, solve $\frac{d2y}{dt^2} + \frac{6dy}{dt} + 5y = e^{-2t}$ given that y = 0,

$\frac{dy}{dt} = 1$

when t = 0.

18. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$.

SECTION – C

Answer any TWO questions (2 x 20 = 40 marks)

19. a) State and prove the first fundamental therorem of integral calculus.

b) The probability density function of the random variable X is $f(x) = \frac{1}{2\theta} \exp\left(-\frac{(x-\theta)}{\theta}\right)$, $-\infty < x < \infty$. Find M.G.F. of X and alsdo find E(X) and var (X). (10 + 10 = 20)

20. a) Prove that the improper integral $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ is convergent.

b) Prove that (i) $\left(n + \frac{1}{2}\right) = \frac{1.3.5...(2n-1)}{2^n} \sqrt{\pi}$ and (ii) (n+1) = n (n).

21. a) Solve the equation $\frac{d2y}{dx^2} - y = (1 + x^2)\, e^x + x sinx$.

b) The joing p.d.f. of the random variables X and Y is given by f (x,y) =

$e^{-(x+y}, \quad x > 0\ y > 0$

$0\,, \quad other\ wise$.

Find the Cov (X,Y) .

22. a) Show that the following system of equations is consistent and hence solve them.

$x - 3y - 8Z = = 10\,; \quad 3x + y - 4Z = 0; \quad 2x + 5y + 6Z = 13.$

b) State and prove Cayley – Hamilton theorem.